

## 7.4 Part 1 Solving Polynomial Equations

### Factoring Refresher

- GCF
- Difference of Squares
- Difference of Cubes
- Sum of Cubes
- Sum/Product Chart
- Grouping

### Solving Refresher

- Set each factor = 0
- Quadratics:
  - > Factor
  - > Complete the Square
  - > Quadratic Formula
  - > Take Square Roots

**solutions = roots = x-intercepts = zeros**

### Example 1

Solve  $2x^3 - 7x^2 + 3x = 0$ .

$$x(2x^2 - 7x + 3) = 0$$

$$x(x-3)(2x-1) = 0$$

sum -7 product 6

$$\frac{-3}{1} \quad \frac{-6}{2} \quad \frac{-1}{2}$$

$$x = 0$$

$$x - 3 = 0$$

$$2x - 1 = 0$$

$$x = 3$$

$$x = \frac{1}{2}$$

**Example 2**

$$\text{Solve } 5x^3 + 4x = 12x^2.$$

$$\begin{array}{r} -12x^2 \quad -12x^2 \\ \hline \end{array}$$

$$5x^3 - 12x^2 + 4x = 0$$

$$x(5x^2 - 12x + 4) = 0$$

$$x(x-2)(5x-2) = 0$$

$$x = 0, 2, \frac{2}{5}$$

$$\begin{array}{r} \text{sum } -12 \quad \text{prod } 20 \\ \hline \frac{-2}{1} \quad \frac{-10}{5} \quad \frac{-2}{5} \end{array}$$

**Example 3**

$$\text{Solve } x^4 - 4x^2 + 3 = 0.$$

$$(x^2 - 3)(x^2 - 1) = 0$$

$$(x^2 - 3)(x+1)(x-1) = 0$$

$$x^2 - 3 = 0$$

$$\sqrt{x^2} = \sqrt{3}$$

$$x = \pm\sqrt{3}, -1, 1$$

$$\begin{array}{r} \text{sum } -4 \quad \text{product } 3 \\ \hline \frac{-3}{1} \quad \frac{-1}{1} \end{array}$$

**Example 4**Solve  $x^4 + 7x^2 - 18 = 0$ .

$$(x^2 + 9)(x^2 - 2) = 0$$

$\frac{\text{sum } 7}{9}$      $\frac{\text{product } -18}{-2}$   
 $\frac{9}{1}$      $\frac{-2}{1}$

$$x^2 + 9 = 0 \qquad x^2 - 2 = 0$$

$$\sqrt{x^2} = \sqrt{-9} \qquad \sqrt{x^2} = \sqrt{2}$$

$x = \pm 3i, \pm \sqrt{2}$

**Example 5**Solve  $x^3 - 3x^2 + 5x = 15$ .

$$\frac{-15 \quad -15}{(x^3 - 3x^2) + (5x - 15)} = 0$$

$$x^2(\cancel{x-3}) + 5(\cancel{x-3}) = 0$$

$$(x-3)(x^2 + 5) = 0$$

$$x^2 + 5 = 0 \rightarrow \sqrt{x^2} = \sqrt{-5}$$

$x = 3, \pm i\sqrt{5}$

**Example 6**  $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

Solve  $x^6 + 16x^3 + 64 = 0$ .

sum 16 product 64

$$(x^3 + 8)(x^3 + 8) = 0$$

$(x)^3 \quad (2)^3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$(x+2)(x^2 - 2x + 4)(x+2)(x^2 - 2x + 4) = 0$$

$$x = -2$$

quad. form.

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)}$$

$$\begin{array}{r} 2 \overline{) 12} \\ 2 \overline{) 6} \\ \underline{\phantom{2} 3} \end{array}$$

$$x = \frac{2 \pm \sqrt{-12}}{2}$$

$$x = \frac{2 \pm 2i\sqrt{3}}{2} = \frac{2}{2} \pm \frac{2i\sqrt{3}}{2} = \boxed{1 \pm i\sqrt{3}}$$

### Example 7

Find the roots of  $P(x) = x^3 - 2x^2 + 2x - 1$  given that

$x - 1$  is a factor.

$$\begin{array}{l} x - 1 = 0 \\ x = 1 \end{array}$$

$$\begin{array}{r} 1 \overline{) 1 \quad -2 \quad 2 \quad -1} \\ \underline{1 \quad -1 \quad 1 \quad 0} \\ 0 \quad 0 \quad 0 \quad 0 \end{array}$$

0 ← rem.

$$(x-1)(x^2 - x + 1) = 0$$

$$x = 1$$

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{-3}}{2}$$

$$x = \frac{1 \pm i\sqrt{3}}{2}$$

**Example 8**Find all zeros of  $P(x) = 2x^3 + 7x^2 + 12x + 9$  given that

$$P\left(-\frac{3}{2}\right) = 0.$$

$$\begin{array}{r|rrrr} -\frac{3}{2} & 2 & 7 & 12 & 9 \\ & \downarrow & -3 & -6 & -9 \\ \hline & 2 & 4 & 6 & 0 \end{array}$$

$$\left(x + \frac{3}{2}\right) (2x^2 + 4x + 6) = 0$$

$$2\left(x + \frac{3}{2}\right) (x^2 + 2x + 3) = 0$$

$$\boxed{x = -\frac{3}{2}}$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(3)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{-8}}{2} = \frac{-2 \pm 2i\sqrt{2}}{2} = \frac{-2}{2} \pm \frac{2i\sqrt{2}}{2}$$

$$\boxed{x = -1 \pm i\sqrt{2}}$$

**Example 9**Solve  $28x - 5x^2 - 3x^3 = 0$ .

$$x(28 - 5x - 3x^2) = 0$$

$$x(7 - 3x)(4 + x) = 0$$

$$\boxed{x = 0, \frac{7}{3}, -4}$$

sum  $-5$  product  $-84$ 

$$\begin{array}{r|l} \frac{-12}{28} & \frac{7}{28} \\ \downarrow & \downarrow \\ \frac{-3}{7} & \frac{1}{4} \end{array}$$