4.1 Exponential Functions

An exponential function has the variable as an exponent.

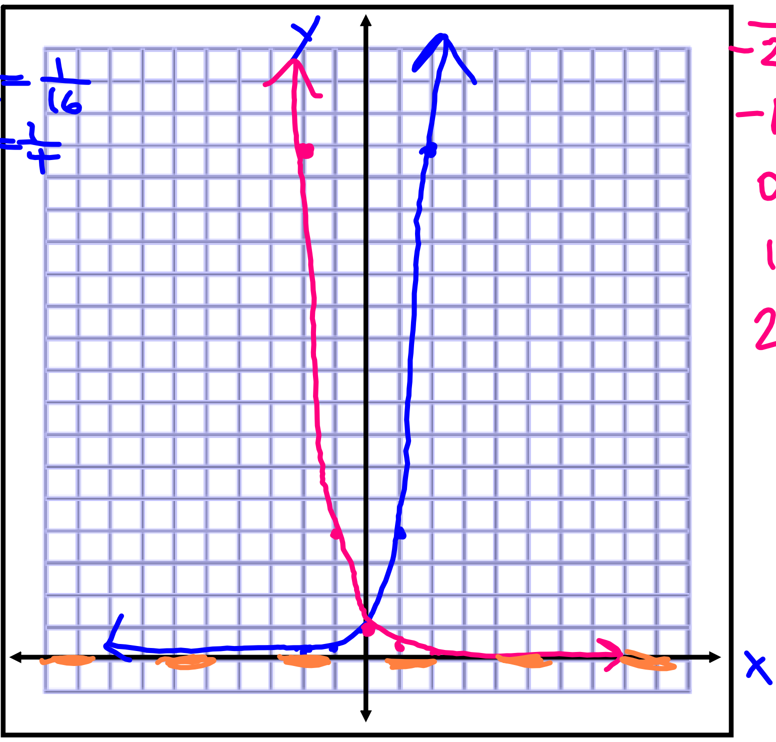
$$f(x) = a \cdot b^x$$

$$b > 0 \text{ and } b \neq 1$$

examples: $f(x) = 2^x$ or $f(x) = \left(\frac{1}{2}\right)^x$

Graph $f(x) = 4^x$ & $f(x) = \left(\frac{1}{4}\right)^x$.

x	y
−2	$4^{-2}=\frac{1}{4^2}=\frac{1}{16}$
−1	$4^{-1}=\frac{1}{4^1}=\frac{1}{4}$
0	$4^0=1$
1	$4^1=4$
2	$4^2=16$



x	y
−2	$\left(\frac{1}{4}\right)^{-2}=\left(\frac{4}{1}\right)^2=16$
−1	$\left(\frac{1}{4}\right)^{-1}=\left(\frac{4}{1}\right)^1=4$
0	$\frac{1}{4}^0=1$
1	$\frac{1}{4}^1=\frac{1}{4}$
2	$\frac{1}{4}^2=\frac{1}{16}$

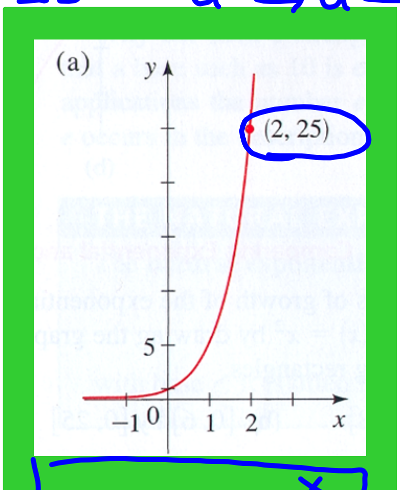
The domain of the parent exponential function is all real numbers and the range is $(0,\infty)$.

The exponential function is a **growth function** if the **base is greater than 1**.

The exponential function is a **decay function** if the **base is between 0 and 1**.

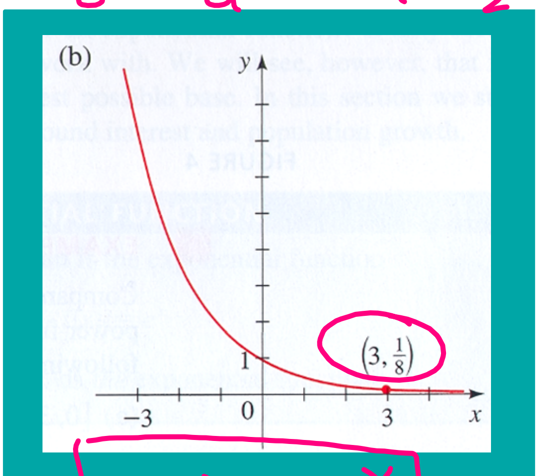
Find the exponential function $f(x) = a^x$ whose graph is given.

$25 = a^2 \rightarrow a = 5$



$\boxed{f(x) = 5^x}$

$\frac{1}{8} = a^3 \rightarrow a = \frac{1}{2}$



$\boxed{f(x) = \frac{1}{2}^x}$

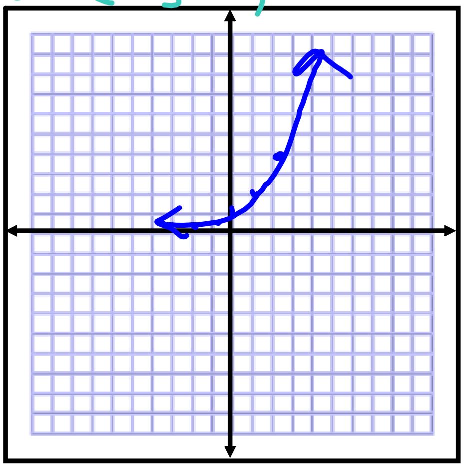
Graph $f(x) = 2^x$.

State the domain and range.

x	y
-2	1/4
-1	1/2
0	1
1	2
2	4

D: $(-\infty, \infty)$

R: $(0, \infty)$

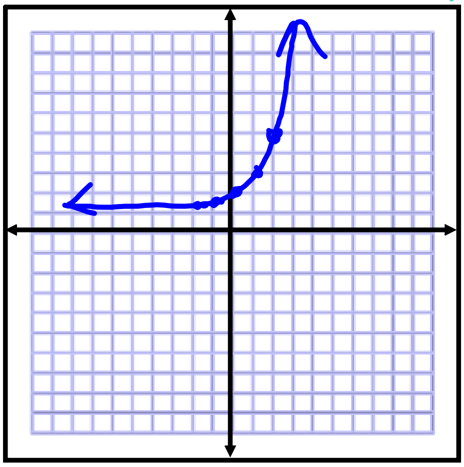


Use the graph of $f(x) = 2^x$ to graph $g(x) = 2^x + 1$. up 1

State the domain & range.

D: $(-\infty, \infty)$

R: $(1, \infty)$



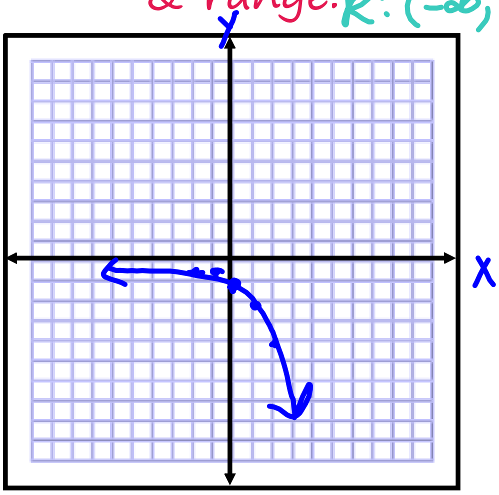
Use the graph of $f(x) = 2^x$ to graph $h(x) = -2^x$.

reflects over x-axis

State the domain & range.

D: $(-\infty, \infty)$

R: $(-\infty, 0)$

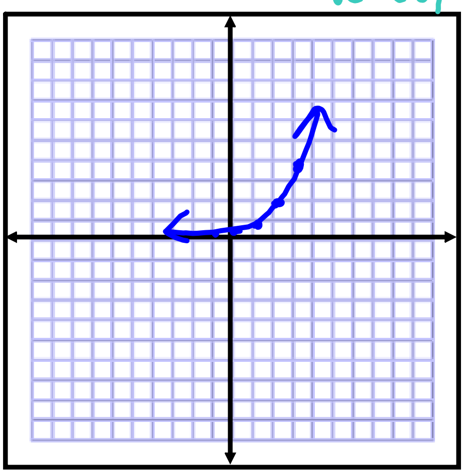


Use the graph of $f(x) = 2^x$ to graph $j(x) = 2^{x-1}$. right 1

State the domain & range.

D: $(-\infty, \infty)$

R: $(0, \infty)$



Compound Interest Formula

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

$A(t)$ = the amount of \$ after t years

P = the amount of \$ invested or borrowed

r = percent as a decimal

t = the number of years

n = the number of times interest is compounded per year

A sum of \$1000 (P) is invested at an interest rate of 12% ($r = .12$) per year. Find the amounts in the account after 3 years ($t = 3$) if interest is compounded annually ($n = 1$), semiannually ($n = 2$), quarterly ($n = 4$), and monthly ($n = 12$).

annually

$A = 1000\left(1 + \frac{.12}{1}\right)^{1 \cdot 3}$

$A \approx \$1404.93$

semiannually

$A = 1000\left(1 + \frac{.12}{2}\right)^{2 \cdot 3}$

$A \approx \$1418.52$

quarterly

$A = 1000\left(1 + \frac{.12}{4}\right)^{4 \cdot 3}$

$A \approx \$1425.76$

monthly

$A = 1000\left(1 + \frac{.12}{12}\right)^{12 \cdot 3}$

$A \approx \$1430.77$

You are offered a temporary job for Donald Trump, starting today and running for the next 30 days. Mr. Trump offers you a choice of two pay packages.

$.01(1+1)^{30}$

#1 He will pay you a penny today. Then for each of the next 30 days that you work, he will double the previous day's pay.

#2 He will pay you $1,000,000 in 30 days.

Which pay package will you take? Why?
How much will you will you be paid?

Exponential Growth Equation: $y = a(1 + r)^t$

r is the percent increase, written as a decimal

1 + r is the growth factor

EXAMPLE 1:

In 1996, there were 2573 computer viruses and security incidents. During the next 7 years, the number of security incidents increased by about 92% each year. Write the exponential equation that models the number of incidents after t years.

$y = 2573(1+.92)^t$

$y = 2573(1.92)^t$

EXAMPLE 2:

In the exponential equation
$y = 527\,(1.39)^{x}$, identify

a) the initial amount, 527

b) the growth factor, 1.39

c) the percent increase. 39%

EXAMPLE 3:

The U.S. population was 248,718,301 in 1990. The projected growth rate is 8% per decade. Write the exponential equation that models the population growth.

$$y = 248{,}718{,}301(1.08)^{\frac{t}{10}}$$

EXAMPLE 4:

In 1970, the population of Kern County, California was about 330,000. From 1970 to 2000, the county population grew at an average annual rate of 2.4%.

a) Write an exponential equation that models the population of Kern County t years after 1970.

b) About how many people lived in Kern County in 1990? $t=20$

a) $y = 330{,}000(1+.024)^t$

b) $y = 330{,}000(1.024)^{20}$

$y \approx 530{,}290$ people

Exponential Decay Equation: $y = a(1-r)^t$

r is the percent decrease, written as a decimal

1 - r is the decay factor

EXAMPLE 5:

A new snowmobile costs $4200. The value of the snowmobile decreases by 10% each year. Write the exponential equation that models the snowmobile's value after t years.

$y = 4200(1-.10)^t$

$y = 4200(.90)^t$

EXAMPLE 6:

In the exponential equation
$y = 200\,(0.71)^{x}$, identify
a) the initial amount, 200
b) the decay factor, $1-r$ .71
c) the percent decrease. 29%

$$1-r=.71$$
$$-1 \qquad -1$$
$$\frac{-r}{-1}=\frac{-.29}{-1}$$
$$r=.29$$

EXAMPLE 7:

A boat depreciates 14% each year. Suppose you paid $25000 for a new boat. Write the exponential decay equation that models the value of the boat after t years.

$$y=25{,}000\,(1-.14)^{t}$$
$$y=25{,}000\,(.86)^{t}$$

EXAMPLE 8:

A new car costs \$32,000. The value of the car decreases by 15% each year.

a) Write an exponential equation that models the value of the car after t years.

b) Estimate the value of the car after 4 years.

a) $y = 32{,}000(1-.15)^t$

b) $y = 32{,}000(.85)^4$

$y \approx \$16{,}704.20$

The present value of a sum of money is the amount that must be invested now, at a given rate of interest, to produce the desired sum at a later date.

EXAMPLE 9: $P = ?$ $A(t)$

Find the present value of \$8,000 if interest is paid at a rate of 6% per year, compounded quarterly, for 5 years.

$r = .06$ $n = 4$

$t = 5$

$$\frac{8000}{\left(1+\frac{.06}{4}\right)^{20}} = \frac{P\cancel{\left(1+\frac{.06}{4}\right)^{4\cdot 5}}}{\cancel{\left(1+\frac{.06}{4}\right)^{20}}}$$

$$\boxed{\$5939.76 \approx P}$$

EXAMPLE 10: $P = ?$

$A(t)$

Find the present value of $25,000 if interest is paid at a rate of 4% per year, compounded monthly, for 7 years.

$r = .04$

$n = 12$ $t = 7$

$$\frac{25{,}000}{\left(1+\frac{.04}{12}\right)^{84}} = \frac{P\cancel{\left(1+\frac{.04}{12}\right)^{12 \cdot 7}}}{\cancel{\left(1+\frac{.04}{12}\right)^{84}}}$$

$$P \approx \$18903.39$$